Statistics

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Statistics

Now we are going to cover how to perform a variety of basic statistical tests in R.

- Correlation
- T-tests
- Linear Regression
- Logistic Regression
- Proportion tests
- Chi-squared
- Fisher's Exact Test

Note: We will be glossing over the statistical theory and "formulas" for these tests. There are plenty of resources online for learning more about these tests, as well as dedicated Biostatistics series at the School of Public Health

Correlation

`cor()` performs correlation in R

```
cor(x, y = NULL, use = "everything",
    method = c("pearson", "kendall", "spearman"))
```

Like other functions, if there are NAs, you get NA as the result. But if you specify use only the complete observations, then it will give you correlation on the non-missing data.

```
> circ = read.csv("http://www.aejaffe.com/summerR_2016/data
+                header=TRUE,as.is=TRUE)
> cor(circ$orangeAverage, circ$purpleAverage)
```

```
[1] NA
```

```
> cor(circ$orangeAverage, circ$purpleAverage, use="complete
```

```
[1] 0.9195356
```

Correlation

You can also get the correlation between matrix columns

```
> signif(cor(circ[,grep("Average",names(circ))],
+              use="complete.obs"),3)
```

```
              orangeAverage purpleAverage greenAverage bann
orangeAverage         1.000         0.908        0.840
purpleAverage         0.908         1.000        0.867
greenAverage          0.840         0.867        1.000
bannerAverage         0.545         0.521        0.453
```

Correlation

You can also get the correlation between matrix columns
Or between columns of two matrices, column by column.

```
> signif(cor(circ[,3:4],circ[,5:6], use="complete.obs"),3)
```

```
                  orangeAverage purpleBoardings
orangeBoardings           0.998           0.922
orangeAlightings          0.998           0.926
```

Correlation

You can also use cor.test() to test for whether correlation is significant (ie non-zero). Note that linear regression may be better, especially if you want to regress out other confounders.

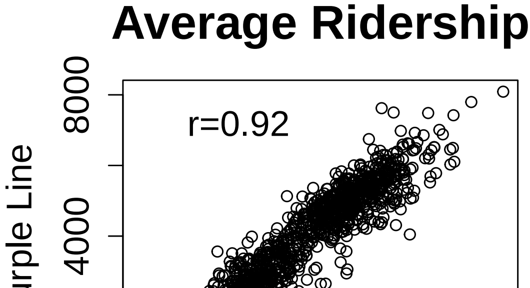
```
> ct= cor.test(circ$orangeAverage,
+      circ$purpleAverage, use="complete.obs")
> ct
```

```

	Pearson's product-moment correlation

data:  circ$orangeAverage and circ$purpleAverage
t = 73.656, df = 991, p-value < 2.2e-16
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.9093438 0.9286245
sample estimates:
      cor
0.9195356
```

Correlation

Note that you can add the correlation to a plot, via the legend() function.

```
> plot(circ$orangeAverage, circ$purpleAverage,
+      xlab="Orange Line", ylab="Purple Line",
+      main="Average Ridership",cex.axis=1.5,
+      cex.lab=1.5,cex.main=2)
> legend("topleft", paste0("r=", signif(ct$estimate,3)),
+        bty="n",cex=1.5)
```



Correlation

For many of these testing result objects, you can extract specific slots/results as numbers, as the ct object is just a list.

```
> # str(ct)
> names(ct)
```

```
[1] "statistic"   "parameter"   "p.value"     "estimate"
[6] "alternative" "method"      "data.name"   "conf.int"
```

```
> ct$statistic
```

```
       t
73.65553
```

```
> ct$p.value
```

```
[1] 0
```

T-tests

The T-test is performed using the `t.test()` function, which essentially tests for the difference in means of a variable between two groups.
In this syntax, x and y are the column of data for each group.

```
> tt = t.test(circ$orangeAverage, circ$purpleAverage)
> tt
```

```

    Welch Two Sample t-test

data:  circ$orangeAverage and circ$purpleAverage
t = -17.076, df = 1984, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equ
95 percent confidence interval:
 -1096.7602  -870.7867
sample estimates:
mean of x mean of y
 3033.161  4016.935
```

T-tests

`t.test` saves a lot of information: the difference in means `estimate`, confidence interval for the difference `conf.int`, the p-value `p.value`, etc.

```
> names(tt)
```

```
[1] "statistic"   "parameter"   "p.value"     "conf.int"
[6] "null.value"  "alternative" "method"      "data.name"
```

T-tests

You can also use the 'formula' notation. In this syntax, it is `y ~ x`, where `x` is a factor with 2 levels or a binary variable and `y` is a vector of the same length.

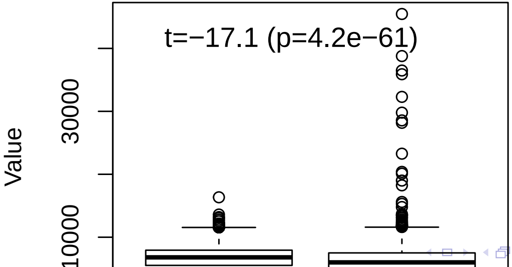
```
> http_data_dir = "http://www.aejaffe.com/summerR_2016/data
> cars = read.csv(paste0(http_data_dir, "kaggleCarAuction.c
+                        as.is=TRUE)
> tt2 = t.test(VehBCost~IsBadBuy, data=cars)
> tt2$estimate
```

```
mean in group 0 mean in group 1
       6797.077        6259.274
```

T-tests

You can add the t-statistic and p-value to a boxplot.

```
> boxplot(VehBCost~IsBadBuy, data=cars,
+         xlab="Bad Buy",ylab="Value")
> leg = paste("t=", signif(tt$statistic,3),
+     " (p=",signif(tt$p.value,3),")",sep="")
> legend("topleft", leg, cex=1.2, bty="n")
```



Linear Regression

Now we will briefly cover linear regression. I will use a little notation here so some of the commands are easier to put in the proper context.

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

where:

- y_i is the outcome for person i
- α is the intercept
- β is the slope
- x_i is the predictor for person i
- ε_i is the residual variation for person i

Linear Regression

The `R` version of the regression model is:

```
y ~ x
```

where:

- y is your outcome
- x is/are your predictor(s)

Linear Regression

For a linear regression, when the predictor is binary this is the same as a t-test:

```
> fit = lm(VehBCost~IsBadBuy, data=cars)
> fit
```

```
Call:
lm(formula = VehBCost ~ IsBadBuy, data = cars)

Coefficients:
(Intercept)     IsBadBuy
     6797.1       -537.8
```

'(Intercept)' is α
'IsBadBuy' is β

Linear Regression

The `summary` command gets all the additional information (p-values, t-statistics, r-square) that you usually want from a regression.

```
> sfit = summary(fit)
> print(sfit)
```

```
Call:
lm(formula = VehBCost ~ IsBadBuy, data = cars)

Residuals:
   Min     1Q Median     3Q    Max
 -6258  -1297    -27   1153  39210

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 6797.077      6.953  977.61   <2e-16 ***
IsBadBuy    -537.803     19.826  -27.13   <2e-16 ***
---
```

Linear Regression

The coefficients from a summary are the coefficients, standard errors, t-statistcs, and p-values for all the estimates.

```
> names(sfit)
```

```
 [1] "call"          "terms"         "residuals"     "coeff
 [5] "aliased"       "sigma"         "df"            "r.squ
 [9] "adj.r.squared" "fstatistic"    "cov.unscaled"
```

```
> sfit$coef
```

```
             Estimate Std. Error   t value     Pr(>|t|)
(Intercept) 6797.0774   6.952728 977.61299  0.00000e+00
IsBadBuy    -537.8033  19.825525 -27.12681 3.01661e-161
```

Linear Regression

We'll look at vehicle odometer value by vehicle age:

```
fit = lm(VehOdo~VehicleAge, data=cars)
print(fit)
```

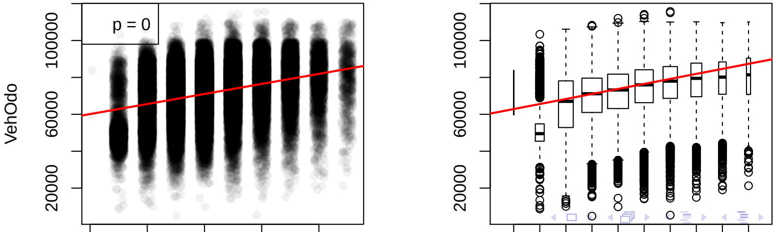
```
##
## Call:
## lm(formula = VehOdo ~ VehicleAge, data = cars)
##
## Coefficients:
## (Intercept)   VehicleAge
##       60127         2723
```

Linear Regression

We can visualize the vehicle age/odometer relationshp using scatter plots or box plots (with regression lines). The function `abline` will plot the regresion line on the plot.

Linear Regression

```
> library(scales) # we need this for the alpha command - mo
> par(mfrow=c(1,2))
> plot(VehOdo ~ jitter(VehicleAge,amount=0.2), data=cars, p
+       col = alpha("black",0.05), xlab="Vehicle Age (Yrs)")
> abline(fit, col="red",lwd=2)
> legend("topleft", paste("p =",summary(fit)$coef[2,4]))
> boxplot(VehOdo ~ VehicleAge, data=cars, varwidth=TRUE)
> abline(fit, col="red",lwd=2)
```



Linear Regression

Note that you can have more than 1 predictor in regression models.The interpretation for each slope is change in the predictor corresponding to a one-unit change in the outcome, holding all other predictors constant.

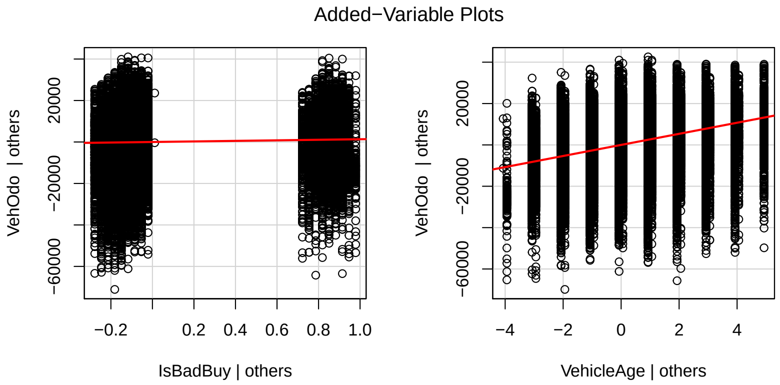
```
> fit2 = lm(VehOdo ~ IsBadBuy + VehicleAge, data=cars)
> summary(fit2)
```

```
Call:
lm(formula = VehOdo ~ IsBadBuy + VehicleAge, data = cars)

Residuals:
   Min      1Q  Median      3Q     Max
-70856   -9490    1390   10311   41193

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 60141.77     134.75  446.33   <2e-16 ***
```

Linear Regression

Added-Variable plots can show you the relationship between a variable and outcome after adjusting for other variables. The function `avPlots` from the `car` package can do this:

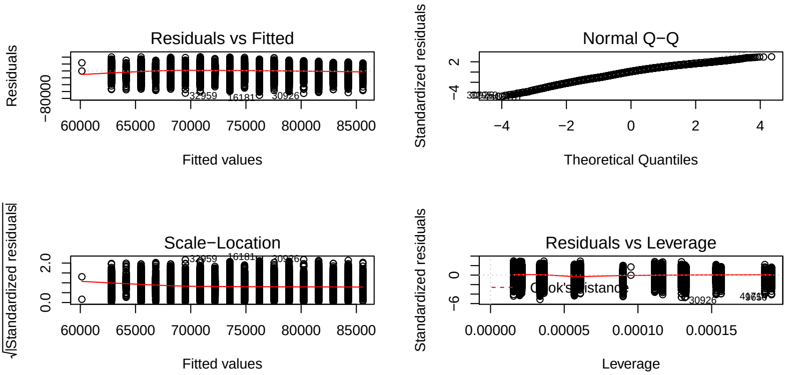
```
> library(car)
> avPlots(fit2)
```



Linear Regression

Plot on an `lm` object will do diagnostic plots. Residuals vs. Fitted should have no discernable shape (the red line is the smoother), the qqplot shows how well the residuals fit a normal distribution, and Cook's distance measures the influence of individual points.

```
> par(mfrow=c(2,2))
> plot(fit2, ask= FALSE)
```



Linear Regression

Factors get special treatment in regression models - lowest level of the factor is the comparison group, and all other factors are relative to its values.

```
> fit3 = lm(VehOdo ~ factor(TopThreeAmericanName), data=car
> summary(fit3)
```

```
Call:
lm(formula = VehOdo ~ factor(TopThreeAmericanName), data =

Residuals:
   Min     1Q Median     3Q    Max
-71947  -9634   1532  10472  45936

Coefficients:
                                 Estimate Std. Error t val
(Intercept)                      68248.48      92.98 733.9
factor(TopThreeAmericanName)FORD  8523.49     158.35  53.8
```

Logistic Regression and GLMs

Generalized Linear Models (GLMs) allow for fitting regressions for non-continous/normal outcomes. The `glm` has similar syntax to the `lm` command. Logistic regression is one example.

```
> glmfit = glm(IsBadBuy ~ VehOdo + VehicleAge, data=cars, f
> summary(glmfit)
```

```
Call:
glm(formula = IsBadBuy ~ VehOdo + VehicleAge, family = binc
    data = cars)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.9943  -0.5481  -0.4534  -0.3783   2.6318

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -3.778e+00  6.381e-02 -59.211   <2e-16 ***
```

Logistic Regression

Note the coefficients are on the original scale, we must exponentiate them for odds ratios:

```
> exp(coef(glmfit))
```

```
(Intercept)      VehOdo  VehicleAge
 0.02286316  1.00000834  1.30748911
```

Proportion tests

prop.test() can be used for testing the null that the proportions (probabilities of success) in several groups are the same, or that they equal certain given values.

```
prop.test(x, n, p = NULL,
          alternative = c("two.sided", "less", "greater"),
          conf.level = 0.95, correct = TRUE)
```

```
> prop.test(x=15, n =32)
```

```

    1-sample proportions test with continuity correction

data:  15 out of 32, null probability 0.5
X-squared = 0.03125, df = 1, p-value = 0.8597
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.2951014 0.6496695
sample estimates:
```

Chi-squared tests

`chisq.test()` performs chi-squared contingency table tests and goodness-of-fit tests.

```
chisq.test(x, y = NULL, correct = TRUE,
           p = rep(1/length(x), length(x)), rescale.p = FAI
           simulate.p.value = FALSE, B = 2000)
```

```
> tab = table(cars$IsBadBuy, cars$IsOnlineSale)
> tab
```

```
        0     1
  0 62375  1632
  1  8763   213
```

Chi-squared tests

You can also pass in a table object (such as tab here)

```
> cq=chisq.test(tab)
> cq
```

```

    Pearson's Chi-squared test with Yates' continuity corre

data:  tab
X-squared = 0.92735, df = 1, p-value = 0.3356
```

```
> names(cq)
```

```
[1] "statistic" "parameter" "p.value"   "method"    "data.n
[7] "expected"  "residuals" "stdres"
```

```
> cq$p.value
```

```
[1] 0.3355516
```

Chi-squared tests

Note that does the same test as `prop.test`, for a 2x2 table.

```
> chisq.test(tab)


	Pearson's Chi-squared test with Yates' continuity corre

data:  tab
X-squared = 0.92735, df = 1, p-value = 0.3356
```

```
> prop.test(tab)


	2-sample test for equality of proportions with continu
	correction

data:  tab
X-squared = 0.92735, df = 1, p-value = 0.3356
alternative hypothesis: two.sided
```

Fisher's Exact test

fisher.test() performs contingency table test using the hypogeometric distribution (used for small sample sizes).

```
fisher.test(x, y = NULL, workspace = 200000, hybrid = FALSE
            control = list(), or = 1, alternative = "two.si
            conf.int = TRUE, conf.level = 0.95,
            simulate.p.value = FALSE, B = 2000)
```

```
> fisher.test(tab)
```

```
    Fisher's Exact Test for Count Data

data:  tab
p-value = 0.3324
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
 0.8001727 1.0742114
sample estimates:
```

Probability Distributions

Sometimes you want to generate data from a distribution (such as normal), or want to see where a value falls in a known distribution. `R` has these distibutions built in:

- Normal
- Binomial
- Beta
- Exponential
- Gamma
- Hypergeometric
- etc

Probability Distributions

Each has 4 options:

- `r` for random number generation [e.g. `rnorm()`]
- `d` for density [e.g. `dnorm()`]
- `p` for probability [e.g. `pnorm()`]
- `q` for quantile [e.g. `qnorm()`]

```
> rnorm(5)
```

```
[1] -1.1038184  2.0441939 -0.3254349  0.1542818 -0.1004937
```

Sampling

The `sample()` function is pretty useful for permutations

```
> sample(1:10, 5, replace=FALSE)
```

```
[1]  5 10  4  6  7
```

Sampling

Also, if you want to only plot a subset of the data (for speed/time or overplotting)

```
> samp.cars <- cars[ sample(nrow(cars), 10000), ]
> plot(VehOdo ~ jitter(VehBCost,amount=0.3), data= samp.car
```